

Econ 301 / Fall 2005
Problem Session 5

Problem 1. *Revealed Preference (MWG 2.F.3)*

You are given the following partial information about a consumer's purchases. He consumes only two goods.

	Year 1		Year 2	
	Quantity	Price	Quantity	Price
Good 1	100	100	120	100
Good 2	100	100	?	80

- a. Over what range of quantities of good 2 consumed in year 2 would you conclude that his behavior is inconsistent (i.e., in contradiction with the weak axiom)?

Now, assume that the weak axiom is satisfied. Over what range of quantities of good 2 consumed in year 2 would you conclude:

- b. That the consumer's consumption bundle in year 1 is revealed preferred to that in year 2?
- c. That the consumer's consumption bundle in year 2 is revealed preferred to that in year 1?
- d. That there is insufficient information to justify a, b, and/or c?
- e. That good 1 is an inferior good (at some price) for this consumer?
- f. That good 2 is an inferior good (at some price) for this consumer?

Problem 2. *Duality in Production*

Consider an industry where output is produced using only capital k and labor l . Let w , r , and q denote the wage and the rental rate of capital and the quantity of output respectively. Let $a, b > 0$. Determine the production function $y(k, l)$ corresponding to the following cost functions:

- a. $c(w, r, q) = \frac{rw}{r+w}q^2$
- b. $c(w, r, q) = (ar + bw)\sqrt{q}$

Problem 3. *Car Factory (Spring 2003 Qualifier)*

Assume that Car Factory (CF) produces cars using unskilled labor (l), robots (r), and skilled labor (h). One robot is a perfect substitute for one unskilled worker. In addition, in order to produce q cars, CF needs at least q skilled workers and at least q^z robots and unskilled workers combined.

- a. Write down the production function of CF on the basis of the above information.
- b. For what values of z does this technology exhibit decreasing, constant, or increasing returns to scale? Explain your answer intuitively.
- c. Show that if $z = 2$, then the firm's cost function has the following form:

$$c(w_l, w_h, w_r, q) = q [w_h + q \min(w_l, w_r)],$$

where w_l and w_h are wage rates for skilled and unskilled labor respectively and w_r is the price per unit of time (i.e., the rental rate) of a robot.

- d. Assume that the demand for cars can be described by the following linear function $d(p) = 11 - p$ and that Car Factory takes prices as given. Assume that $w_l = w_r = 1$ and $w_h = 2$. Derive the optimal supply and factor demands. Calculate the profit of this company.

Now, assume that there are two countries where firms use this same technology. In country A, unions are strong and the firm, therefore, has to sign long-term contracts with unskilled workers. In country B, the firm can adjust its labor demand instantaneously. Assume that in both countries there are 16 unskilled workers employed but because of technological improvements the price of robots drops from 1 to 1/2. Everything else is as in part d.

- e. Analyze the optimal supply and factor demands immediately after the price changes in country A and B. Compare the profits in country A and B to the ones derived in part d. Explain the economic intuition.
- f. Compare the welfare changes of consumers and producers in the two countries.
- g. Explain why these results may be misleading from the general equilibrium point of view.