Problem 1. *Moe's Tavern*

There is a continuum of residents of size $M = 1$ living in Springfield, distributed uniformly along the unit interval. Everyone in Springfield has an inelastic unit demand for one can of Duff beer which they each value at $v = 13$. Everyone in Springfield also hates walking and has a quadratic travel cost function so that walking a distance of d causes disutility d^2. Unfortunately, Moe's tavern, on the west side of town (at 0) is currently the only place to drink Duff. For example, Homer, who lives at $z \in [0, 1]$, has to pay a travel cost of $(2z)^2$ (walking to Moe's and back home). [This is a variation on the Linear City Model with quadratic travel cost.]

a. Moe currently has a monopoly on Duff in Springfield, despite his poor location, and he can set any price p_m he wants. His cost per can of Duff is $c = 1$. What is Moe's profit function and what is the monopoly price? How much profit does he earn?

b. What fraction of Springfield (σ_m) comes to Moe's under the monopoly price? What is the value of consumer surplus?

Now, Willie's Tavern opens on the east side of town (at 1) and begins selling Duff, also at a cost of $c = 1$ per can. Moe and Willie compete by setting prices p_0 and p_1.

c. What is Moe's profit function now (as a function of Willie's price p_1)? Derive his best response function $BR_0(p_1)$. Repeat this process for Willie.

d. Find the symmetric equilibrium price $p_0 = p_1 = p^*$ and the resulting profit levels.

e. What fraction of the residents go to Moe's and fraction goes to Willie's? What is the resulting value of consumer surplus?

Problem 2. *Two-Player All-Pay Auction*

Consider a symmetric full-information all-pay auction with two bidders. Both bidders are risk-neutral and must pay their own bid regardless of the outcome of the auction. The value of the object to each bidder is $v > 0$. Bids must be non-negative real numbers. Let F be the CDF representing a symmetric, mixed-strategy Nash equilibrium with support $[\underline{b}, \bar{b}]$.

a. What is each bidder's equilibrium expected utility u^*?

b. Suppose one bidder bids b with certainty while his opponent mixes according to F. What is his expected utility $\tilde{u}(b)$?

c. Derive an expression for F and find its support.

d. What is each player's expected bid? What is the expected revenue of the seller?