Econ 302 **2006.02.10**
Problem Session 4 **Jason Blevins**

Problem 1. *Castle Siege*

Two lords and their armies are battling over a castle. Both armies only have enough food to fight for a year. If they fight longer, everyone will die. The lords can each give their armies only one order: the length of time t_i to siege. Let $l = \min\{t_1, t_2\}$ denote the length of the battle. The castle is worth 1 to each lord but fighting for a time t damages both the castle and the armies by t. The utility of each lord i is given by

$$u_i(t_i, t_j) = I_{\{t_i > t_j\}}(1 - l) - l.$$

a. Find a pure strategy Nash equilibrium.

b. Find a symmetric mixed-strategy Nash equilibrium in which both lords randomize according F.

Problem 2. *Midterm 2005*

Consider the following dynamic game of perfect information. An individual (the buyer) wishes to purchase one unit of an indivisible good which he values at $v > 0$. There are two firms (the sellers) that can produce the good at zero cost. The buyer has two period-zero options: he may *quit*, ending the game with payoffs of zero to all players, or he may *visit seller 1* for a price quote. The buyer incurs a small transaction cost of $c \in (0, v/2)$ for each seller he visits. If visited, seller 1 makes the buyer a price offer of $p_1 \in \mathbb{R}_+$. The buyer then has three period-one options. He may *accept seller 1's offer*, yielding a payoff to the buyer of $v - c - p_1$, a payoff to seller 1 of p_1, and a payoff to seller 2 of zero; he may *quit* yielding a payoff to the buyer of $-c$ and payoffs to the sellers of zero; or he may *visit seller 2*. If visited, seller 2 observes p_1 and makes the buyer a price offer $p_2 \in \mathbb{R}_+$. The buyer then has three period-two options. He may *accept buyer i's offer*, yielding a payoff to the buyer of $v - 2c - p_i$, a payoff to seller i of p_i, and a payoff to seller $j \neq i$ of zero for $i = 1, 2$; or he may *quit* yielding a payoff of $-2c$ to the buyer and zero to the sellers.

a. Specify the buyer's SPNE strategy in period two, $s_2^*(p_1, p_2)$.

b. Specify seller 2's SPNE strategy, $p_2^*(p_1)$.

c. Specify the buyer's SPNE strategy in period one, $s_1^*(p_1)$

d. Specify seller 1's SPNE strategy, p_1^*.

e. Specify the buyer's SPNE strategy in period zero, s_0^*.

f. What is the SPNE outcome of this game? Is the SPNE outcome Pareto efficient? If not, why not?

Problem 3. *Midterm 2005*

Consider the repeated game, $\Gamma^1(1)$, that consists of playing the following stage game *twice* without discounting ($\delta = 1$): Let $x \in \{(Q,Q),(Q,C),(C,Q),(C,C)\}$ denote a one-period history (i.e., a subgame).

Player 2

Player 1		Q	C
	Q	(-1, -1)	(-10, 0)
	C	(0, -10)	(-9, -9)

Table 1: Prisoners' Dilemma

a. How many pure strategies does each player have in the repeated game $\Gamma^1(1)$? How many pure-strategy profiles are there in $\Gamma^1(1)$?

b. Fully specify a subgame-perfect Nash equilibrium profile of $\Gamma^1(1)$. How many pure-strategy SPNE profiles are there in $\Gamma^1(1)$?

c. Fully specify a Nash equilibrium profile of $\Gamma^1(1)$ that is *not* subgame perfect. How many Nash equilibrium profiles are there in $\Gamma^1(1)$? (Hint: (C,C) must occur on the equilibrium path in both rounds of play in every NE.)