Epple and Sieg (1999): Estimating Equilibrium Models of Locational Jurisdictions

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Framework

Locations:

- Single metro area with J communities
- Composite public good g, housing prices p
- $\{(g_1, p_1), \ldots, (g_J, p_J)\}$

Households:

- Continuum C of price taking households with costless mobility
- Income y and taste parameter α with distribution $f(\alpha, y)$
- $(\alpha, y) \in C = \mathbb{R}^2_+$
- Consume housing (h), private goods (b), and public goods (g)
- Utility maximization

$$\max_{h,b} \quad U(\alpha, g, h, b) \quad s.t. \quad ph + b = y$$

- Single-crossing property

Equilibrium

- Community budgets are balanced
- All markets clear
- No household wishes to change residence

Indirect Utility

Indirect utility function

$$V(\alpha, g, p, y) = U\left(\alpha, g, h(p, y, \alpha), y - ph(p, y, g, \alpha)\right)$$

"Indirect indifference curves"

$$\left\{(p, g) \in \mathbb{R}^2_+ \mid V(\alpha, g, p, y) = \bar{V}\right\}$$

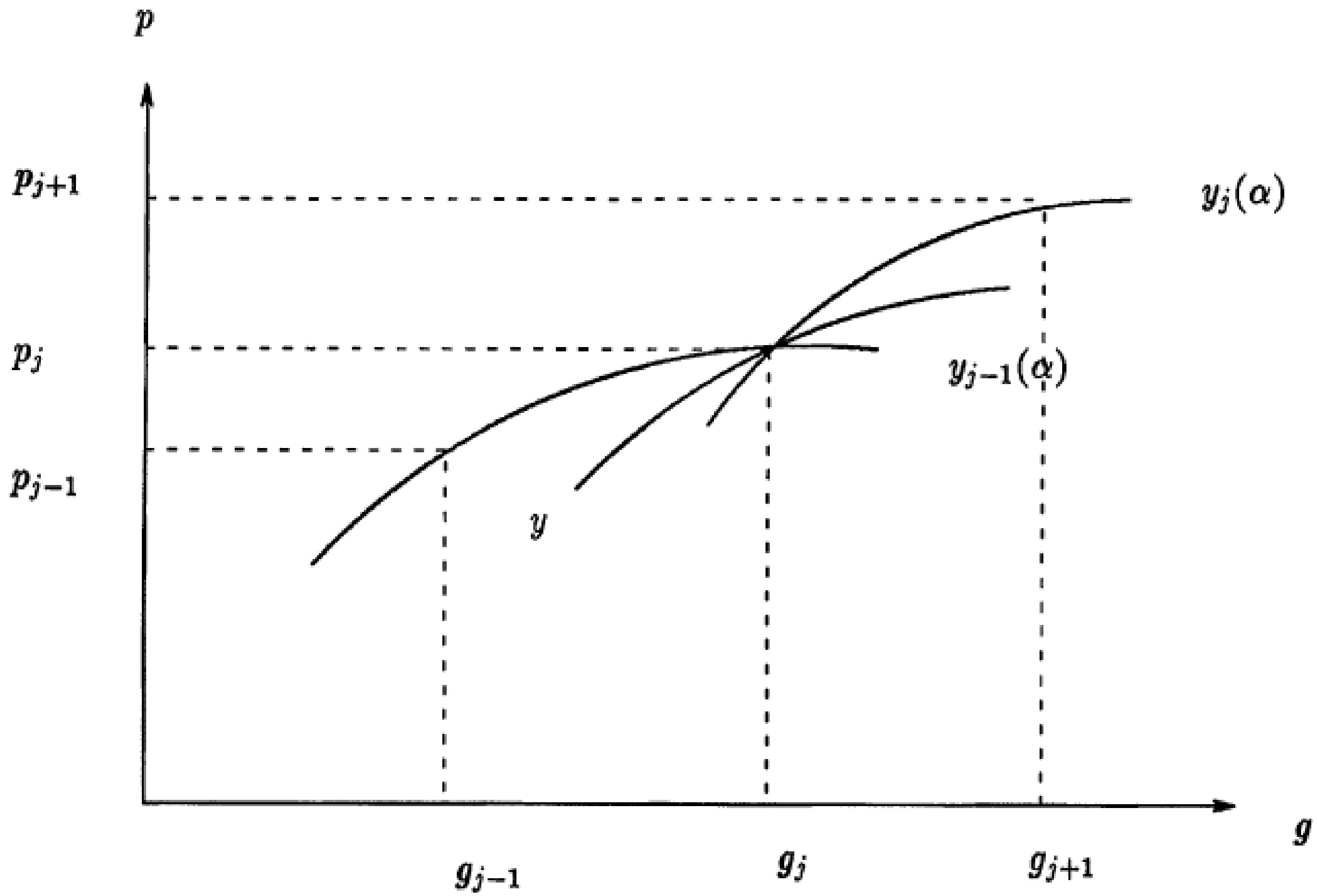
Slope of indirect indifference curves

$$M(\alpha, g, p, y) = \left.\frac{\partial p}{\partial g}\right|_{V=\bar{V}} = -\frac{\partial V(\alpha, g, p, y)/\partial g}{\partial V(\alpha, g, p, y)/\partial p}$$

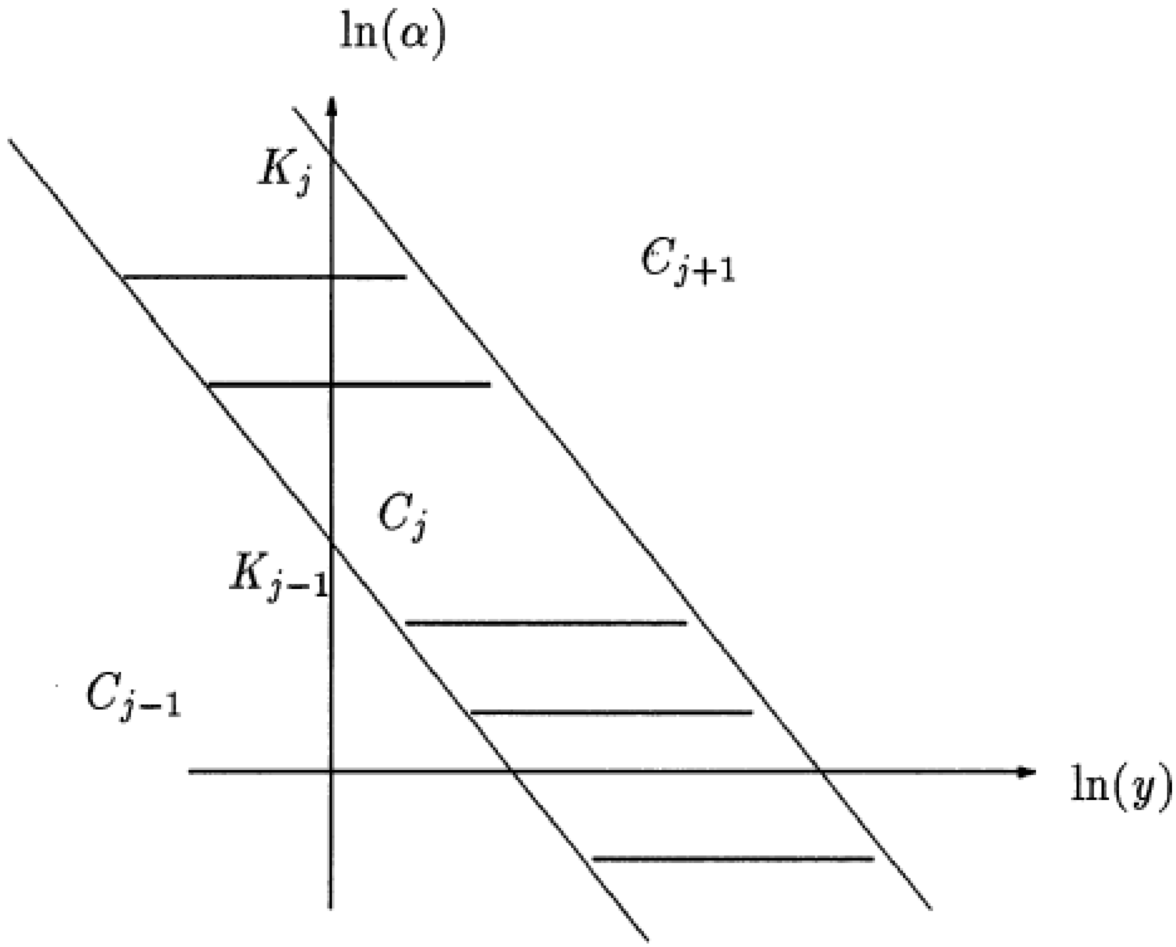
Boundary indifference condition

$$V(\alpha, g_{j-1}, p_{j-1}, y) = V(\alpha, g_j, p_j, y) \quad \Rightarrow \quad y_j(\alpha)$$

Single-crossing property



Boundary Indifference Conditions



Parametric Assumptions

$$V(g,p,y,\alpha) = \left\{ \alpha g^{\rho} + \left[\exp\left(\frac{y^{1-\nu} - 1}{1-\nu} \right) \exp\left(-\frac{Bp^{\eta+1} - 1}{1+\eta} \right) \right]^{\rho} \right\}^{1/\rho}$$

where $\rho < 0$, $\alpha > 0$, $\eta < 0$, $\nu > 0$, and $B > 0$

$$h(p,y) = Bp^{\eta} y^{\nu}$$

$$M(g,p,y,\alpha) = \frac{\alpha g^{\rho-1} \left[\exp\left(\frac{y^{1-\nu}-1}{1-\nu} \right) \right]^{-\rho} \left[\exp\left(-\frac{Bp^{\eta+1}-1}{1+\eta} \right) \right]^{-\rho}}{Bp^{\eta}} > 0$$

(α, y) jointly bivariate log-normal with correlation λ.

Boundary Indifference Conditions

$$\ln(\alpha) - \rho\left(\frac{y^{1-\nu} - 1}{1-\nu}\right) = \ln\left(\frac{Q_{j+1} - Q_j}{g_j^{\rho} - g_{j+1}^{\rho}}\right)$$

where

$$Q_j = \exp\left[-\frac{\rho}{1+\eta}\left(Bp_j^{\eta+1} - 1\right)\right]$$

$$K_0 = -\infty$$

$$K_j = \ln\left(\frac{Q_{j+1} - Q_j}{g_j^{\rho} - g_{j+1}^{\rho}}\right)$$

$$K_J = \infty$$

Income Quantiles

Households living in C_j:

$$P(C_j) = \int_{-\infty}^{\infty} \int_{K_{j-1}+\rho[(y^{1-\nu}-1)/(1-\nu)]}^{K_j+\rho[(y^{1-\nu}-1)/(1-\nu)]} f\left(\ln(\alpha), \ln(y)\right) d\ln(\alpha) d\ln(y)$$

q-th quantile of community j's income distribution $\zeta_j(q)$:

$$\int_{-\infty}^{\ln[\zeta_j(q)]} \int_{K_{j-1}+\rho[(y^{1-\nu}-1)/(1-\nu)]}^{K_j+\rho[(y^{1-\nu}-1)/(1-\nu)]} f\left(\ln(\alpha), \ln(y)\right) d\ln(\alpha) d\ln(y) = qP(C_j)$$

Estimation Strategy

Two-stage method:

1. Match quantiles of the income distributions

$$\theta_1 = \left(\nu, \frac{\rho}{\sigma_\alpha}, \mu_y, \sigma_y, \lambda\right)^\top$$

2. Public-good provision

$$\theta_2 = (\rho, \eta, \gamma, \sigma_\alpha, \mu_\alpha)^\top$$

Matching Quantiles

$$F_j\left[\zeta_j(q,\theta)\right] = q \quad \longleftrightarrow \quad \zeta_j^N(q) = F_{j,N}^{-1}(q)$$

$$e_1^N(\theta) = \begin{pmatrix} \ln[\zeta_1(0.25,\theta)] - \ln[\zeta_1^N(0.25)] \\ \ln[\zeta_1(0.50,\theta)] - \ln[\zeta_1^N(0.50)] \\ \ln[\zeta_1(0.75,\theta)] - \ln[\zeta_1^N(0.75)] \\ \vdots \\ \ln[\zeta_J(0.25,\theta)] - \ln[\zeta_J^N(0.25)] \\ \ln[\zeta_J(0.50,\theta)] - \ln[\zeta_J^N(0.50)] \\ \ln[\zeta_J(0.75,\theta)] - \ln[\zeta_J^N(0.75)] \end{pmatrix}$$

$$\theta_1 = \left(\nu, \frac{\rho}{\sigma_\alpha}, \mu_y, \sigma_y, \lambda\right)^\top$$

$$\theta_1^N = \arg\min_{\theta_1 \in \Theta_1} e_1^N(\theta_1)^\top A_1^N e_1^N(\theta_1)$$

Public good provision

Public good provision index:

$$g_j = x_j^\top \gamma + \epsilon_j$$

Solve for g_{j+1}:

$$g_{j+1}^\rho = g_j^\rho - (Q_{j+1} - Q_j)\exp(-K_j)$$

Rewrite:

$$g_j = \left[g_1^\rho - \sum_{i=2}^{j} (Q_i - Q_{i-1})\exp(-K_i) \right]^{1/\rho}$$

Substitute for g_j:

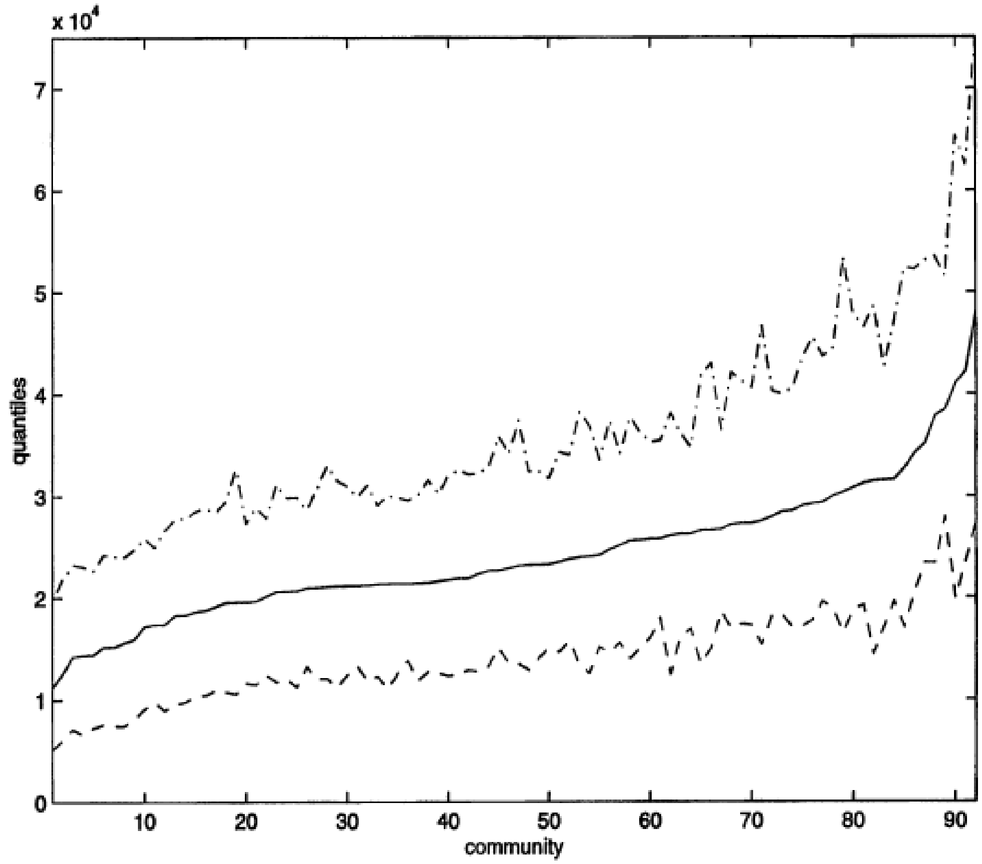
$$\epsilon_j = x_j^\top - \left[g_1^\rho - \sum_{i=2}^{j} (Q_i - Q_{i-1})\exp(-K_i) \right]^{1/\rho}$$

Second-stage parameters:

$$\theta_2 = (\rho, \eta, \gamma, \sigma_\alpha, \mu_\alpha)$$

Data Set

- 1980 Census
- Boston Metro Area
  - Munincipal and school district boundaries coincide
  - 92 communities of various types
  - Substantial income stratification
- Public goods: education expenditures, crime rate



Notation: — median, - - 25% quantile, -. 75% quantile.

Figure 1: Estimated quantiles

First-stage results

Parameters	Estimates
$\mu_{\ln(y)}$	9.790
	(.002)
$\sigma_{\ln(y)}$	.755
	(.004)
λ	-.019
	(.031)
$\rho/\sigma_{\ln(\alpha)}$	-.283
	(.013)
ν	.938
	(.026)
Function value	.0368
Degrees of freedom	271

Table 1: Estimated Parameters: Stage 1

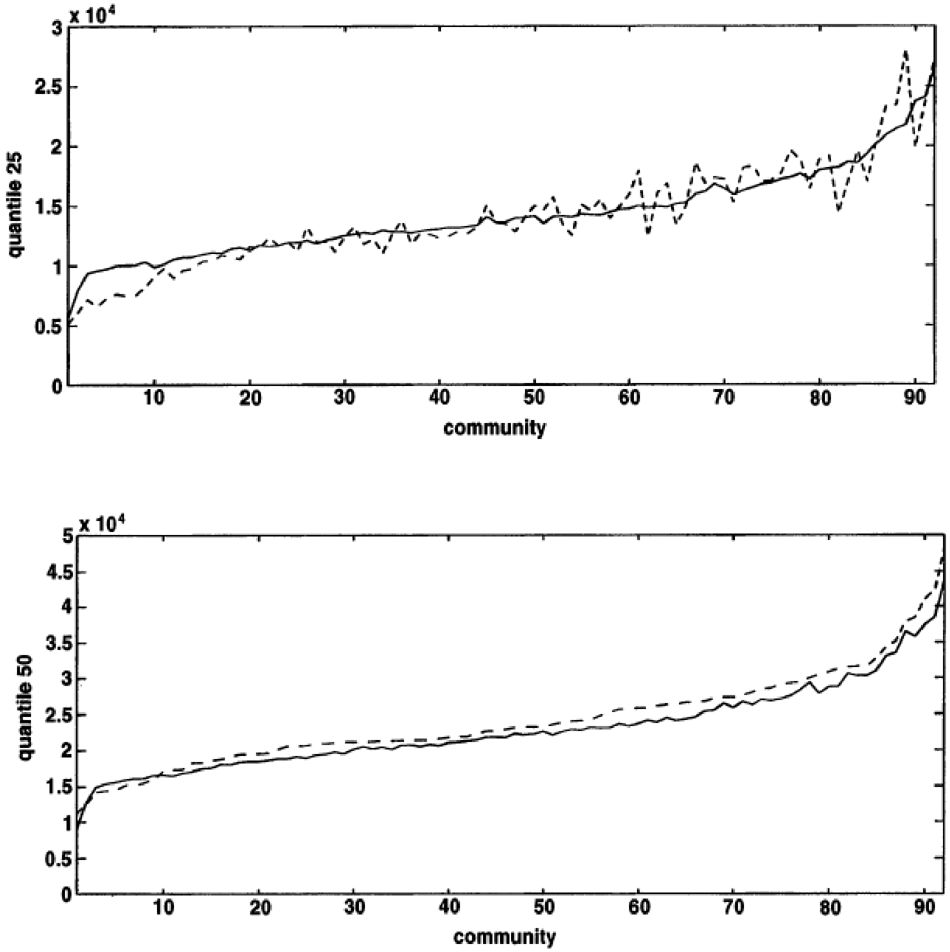


Figure 2: Predicted and estimated quantiles

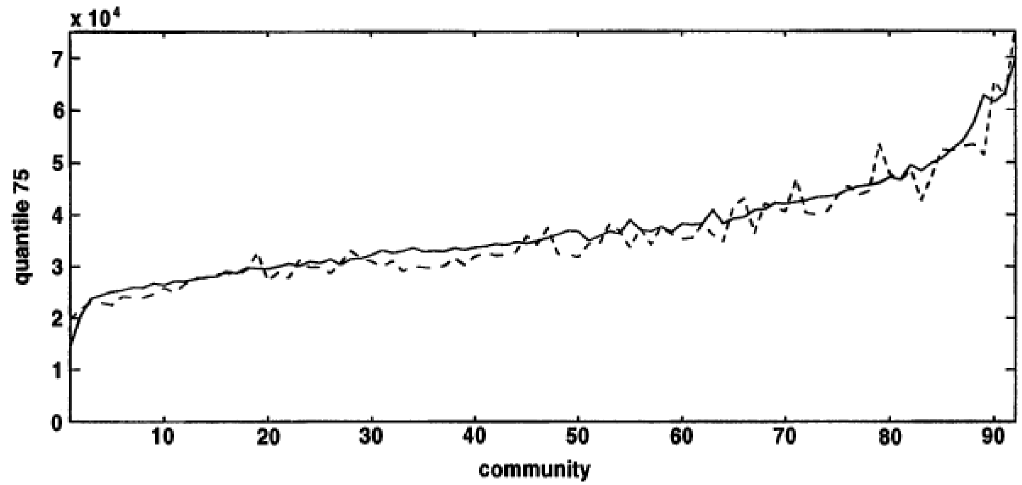
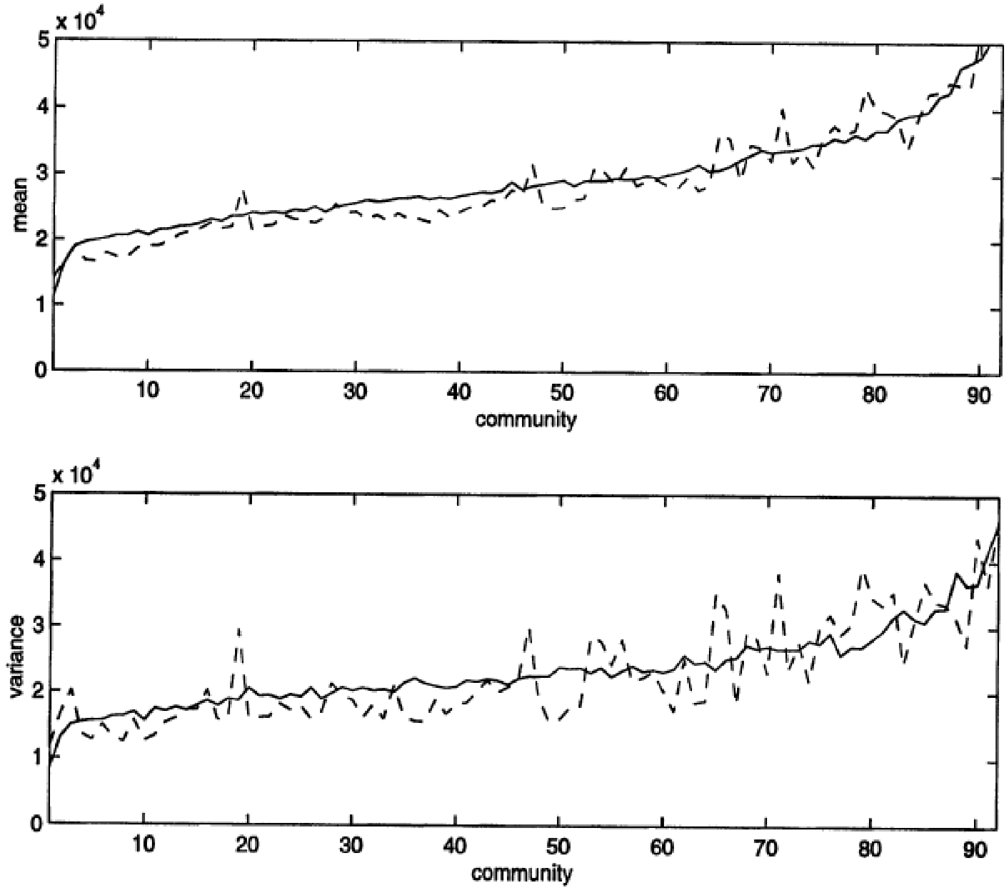
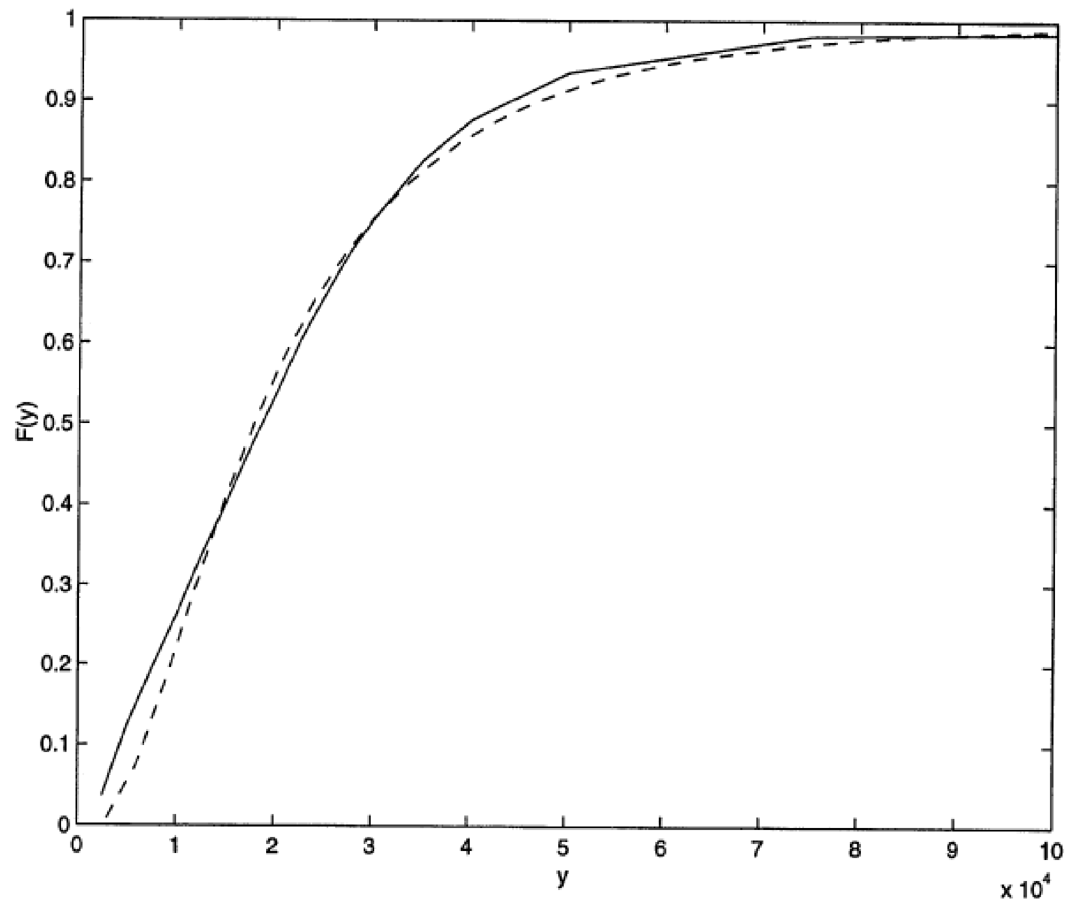


Figure 3: Predicted and estimated quantiles



Notation: - - estimated by the Census, — predicted from our model.

Figure 4: Predicted and estimated moments



Notation: - - estimated by the Census, — predicted from our model.

Figure 5: Income distribution of the BMA

Second-stage results

	NLS		GMM		
	(1)	(2)	(3)	(4)	(5)
γ	-1.95	-1.97	-1.97	-2.08	-2.26
	(1.88)	(1.91)	(4.95)	(4.99)	(1.12)
$\mu_{\ln(\alpha)}$	-2.48	-1.91	-3.11	-2.91	-3.36
	(.65)	(2.87)	(1.80)	(1.38)	(0.73)
$\sigma_{\ln(\alpha)}$	.60	.64	.81	.83	.87
	(.19)	(.58)	(.34)	(.38)	(.24)
ρ	-.17	-.18	-.23	-.23	-.25
	(.05)	(.16)	(.10)	(.11)	(.07)
η	-.30	-.70	-.30	-.50	-.30
		(2.01)			

Table 2: Estimated Parameters: Stage 2