Demand for Differentiated Durable Products: The Case of the U.S. Computer Printer Market

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Introduction

- A dynamic model of demand for differentiated durable goods.
  - Differentiated in many other dimensions in addition to price.
  - Durables yield consumption services over many periods.
  - Quality evolves over time.

- Problems with static discrete choice model:
  - Assumes consumers participate every period.
  - Predict consistent upward trend in sales due to improving quality and falling prices.
  - Cannot account for intertemporal demand substitution.

- The dynamic model is estimated using a two-stage approach.

- The model is applied to the computer printer market to test the hypothesis of forward-looking consumers.

Model

- Discrete time $t = 0, 1, \ldots$.
- Consumers $i = 1, \ldots, M$ each have unit demand over lifetime.
- Consumer i's state (own product or not) is $S_{it} \in \{0, 1\}$. Effective market size at time t is M_t
- Set of products available at time t is J_t.
- If consumer i purchases product j at time t, the payoff is

$$u_{ijt} = f(x_j, y_{jt}; \theta_i) + \varepsilon_{ijt}$$

  otherwise, the consumer obtains payoff c for one period.
- Static product characteristics x_j, dynamic product characteristics y_{jt}, and preferences θ_i.

Assumptions

- Consumers share a common discount factor $\beta \in [0, 1)$.
- Consumer preferences are homogeneous: $\theta_i = \theta$ for all $i = 1, \ldots, M$.
- The terms ε_{ijt} are independent across i and t and follow the GEV distribution with CDF $F_\varepsilon(\varepsilon_1, \ldots, \varepsilon_{J_t}) = \exp\left[-G\left(\exp(-\varepsilon_1), \ldots, \exp(-\varepsilon_{J_t})\right)\right]$.
- Then, for a representative consumer, we can write the payoff as $u_{jt} = \delta_{jt} + \varepsilon_{jt}$.

Consumer's Problem

- First, given realizations of ε_{ijt} at time t, consumers determine which product $j_t^* \in J_t$ would maximize their utility.

- Then, the consumer decides whether to purchase the product or wait until next period. This generates an optimal stopping problem

$$J(I_t) = \max_{\tau} \left[\sum_{k=t}^{\tau-1} \beta^{k-t} c + \beta^{\tau-t} E_t \max_{j \in J_t} u_{j\tau} \right].$$

- Here, $E_t[\cdot] \equiv E[\cdot | I_t]$ denotes the conditional expectation given the information available to *all* consumers at time t.

- Thus, if we knew the distribution of $\max_{j \in J_t} u_{j\tau}$, we could formulate this as a standard dynamic discrete choice problem.

Consumer's Problem: Reformulation

- Define
$$v_t \equiv \max_{j \in J_t} u_{jt}.$$

- Given the GEV assumption, v_t has a Type I Extreme Value distribution with CDF
$$F_v(u; r_t) = \exp\left[-\exp\left(-(u - r_t)\right)\right]$$

- $r_t \equiv \ln G\left(\exp(\delta_{1t}), \ldots, \exp(\delta_{J_t,t})\right)$ is the mode of the distribution and is effectively a scalar sufficient statistic for the distribution of future payoffs.

- Now, we can write the Bellman equation:
$$J(v_t, I_t) = \max\left\{v_t, c + \beta E_t J(v_{t+1}, I_{t+1})\right\}. \tag{1}$$

Supply

- In order to solve the consumer's problem, we need to know how products are evolving.
- This includes prices, attributes, and the set of available products.
- However, we saw before that the distribution of v_t was characterized completely by a scalar variable r_t.
- r_t can be thought of as the value of being a consumer in the market as a whole.
- Rather than try to model the r_t process explicitly, Melnikov assumes that it is a Markov process with transition density $\Phi(r_{t+1}|r_t, \theta_r)$.
- We specify a functional form for Φ, namely that r_t follows a diffusion process $r_{t+1} = \mu(r_t) + \sigma(r_t)\nu_{t+1}$ where the ν_t are *iid* standard Normal.

Diffusion Process

$$r_{t+1} = \mu(r_t) + \sigma(r_t)\nu_{t+1}$$

$\mu(r)$ and $\sigma(r)$ must satisfy the following properties:

1. $\mu(r)$ and $\sigma(r)$ are continuous and differentiable *a.e.*

2. $0 < \sigma(r) < \infty$ for all $r \in \mathbb{R}$.

3. r_t is a weak submartingale: $\mu(r_t) \geq r_t$.

4. $\lim_{n\to\infty} \beta^n \mu^n(r) < \infty$ where $0 \leq \beta < 1$, $\mu^0(r) = \mu(r)$, and $\mu^n(r) = \mu\left(\mu^{n-1}(r)\right)$.

Solving the Consumer's Problem

Under the previous assumptions, we can write 1 as

$$J(v, r) = \max\{v, W(r)\}$$

where v has a Type 1 Extreme Value distribution with mode r and

$$W(r) = c + \beta E[J(v', r')|r]$$

is the reservation utility. This is an optimal stopping problem with stopping set

$$\mathcal{S} = \{v|v \geq W(r)\}.$$

Demand

- The consumer will buy a product at time t if and only if $v_t \geq W(r_t)$.

- Therefore, the probability of postponing is implied by the model:

$$\pi_{0t} = \Pr\left\{S_{i,t+1} = 0 | S_{it} = 0, r_t\right\} = \exp\left\{- \exp\left[- \left(W(r_t) - r_t\right)\right]\right\}.$$

- We can also define the hazard rate of product adoption:

$$h(r_t) = 1 - \pi_{0t}(r_t).$$

- Product-specific purchase probabilities:

$$\pi_{jt}(r_t) = h(r_r)\frac{exp(\delta_{jt})G_j(\cdot)}{G(\cdot)}$$

Hazard Rate

- The reservation utility only affects the choice probabilities through the hazard rate $h(r_t)$.

- The hazard rate is a monotone transformation of $Y(r_t) \equiv W(r_t) - r_t$.

- We can derive a functional equation for $Y(r_t)$.

- Note that $v_t \equiv r_t + \varepsilon$ where ε follows the mode zero extreme value distribution.

- We have $W(r_t) = c + \beta E[r_{t+1} + \max(\varepsilon, W(r_{t+1}) - r_{t+1})|r_t]$ and therefore

$$Y(r_t) = c + \beta E[r_{t+1}|r_t] - r_t + \beta E[\max(\varepsilon, Y(r_{t+1}))|r_t].$$

Hazard Rate: Numerical solutions

- We can solve the previous functional equation numerically:

$$Y(r) = c + \beta\mu(r) - r + \beta \int_{-\infty}^{\infty} M(Y(x))\phi\left(\frac{x - \mu(r)}{\sigma(r)}\right) dx.$$

where

$$M(x) \equiv E \max(z, \varepsilon).$$

- Requires only one dimensional numerical integration.

- Once we know $Y(r)$, we can find the hazard rate and choice probabilities.

Aggregation

- Integrate over the joint distribution of c and β to obtain aggregate demand.
- Assumption: $c_i = c$ and $\beta_i = \beta$ for all i.
- Transition of consumer state $S_{it} \in \{0, 1\}$ follows

$$H(r_t) = \begin{bmatrix} \pi_{0t}(r_t) & r_t \\ q & 1 - q \end{bmatrix}.$$

where q is an exogenous "break-down" probability, allowing consumers to return to the market.

- The participation rate $\mu_t = \Pr(S_{it} = 0)$ evolves according to:

$$\mu_{t+1} = \mu_t \pi_{0t}(r_t) + q(1 - \mu_t).$$

- The implied market shares are

$$s_t = \mu_t h(r_t) \quad \text{and} \quad s_{jt} = \mu_t h(r_t) \frac{\exp(\delta_{jt}) G_j(\cdot)}{G(\cdot)}.$$

Econometric specification

The econometrician observes:

- Q_{jt}: sales of product j in period t.
- M_t: total market size (taken to be number of households).
- x_j: vector of static product attributes.
- y_{jt}: vector of dynamic product attributes. Define the following:
- Total sales of product j: $Q_j = \sum_j Q_{jt}$.
- Absolute market shares: $s^a_{jt} = Q_{jt}/M_t$. Relative market shares: $s^r_{jt} = Q_{jt}/Q_t$.

Estimation: Static Parameters

- Taking the natural log of the ratio of π_{jt} to π_t gives

$$\ln\left(\frac{\pi_{jt}}{\pi_t}\right) = \delta_{jt} - r_t = \xi_j + y_{jt}b - r_t.$$

- We can estimate these parameters using the sample analog s^r_{jt} of $\ln(\pi_{jt}/pi_t)$ via OLS.

- Use estimated time effects $\hat{r}_t$ as estimates of the realizations of the market quality process.

Estimation: Transition Kernel

- Estimate the parameters of the diffusion process.

- Using estimated $\hat{r}_t$'s, we can estimate $\hat{\theta}_r$ using the functional form of the transition kernel $\Phi(\hat{r}_{t+1}|\hat{r}_t;\theta_r)$ via Maximum Likelihood.

Estimation: Dynamic Parameters

- We have $\hat{\theta}_r$, so for any θ_v, we can evaluate $Y(r)$, the hazard function, and the purchase probabilities.

- Let $N_t = \sum_{i=1}^{M_t}(1-S_{it}) = M_t\mu_t$ denote the number of consumers shopping for a new product.

- Then, the expected level of aggregate sales is

$$\hat{Q}_t(\theta_v, q, N_t; \hat{r}_t, \hat{\theta}_r) = N_t h(\hat{r}_t; \hat{\theta}_r, \theta_v). \quad (2)$$

- The number of consumers who are shopping next period equals the number who choose to continue searching this period plus the replacement demand inflow plus any exogenous change in the market size:

$$N_{t+1} = N_t \pi_{0t}(\hat{r}_t; \hat{\theta}_r, \theta_v) + q(M_t - N_t) + (M_{t+1} - M_t). \quad (3)$$

- Thus, the remaining parameters θ_v, q, and μ_0 can be found using 2 and 3 to fit predicted sales to the data: $\hat{Q}_t(\hat{r}_t; \theta_d, \hat{\theta}_r) = Q_t$.

Monte Carlo Results

- $$u_{jt} = \xi_j + \alpha x_{jt} + \varepsilon_{jt} = \delta_{jt} + \varepsilon_{jt}$$

- ε_{jt} are *iid* Type 1 Extreme Value.

- $$r_{t+1} = r_t + \gamma + \sigma_r \nu_{t+1}, \quad \nu_{t+1} \sim N(0,1)$$

  where $r_t = \ln \sum \exp(\xi_j + \alpha x_{jt})$.

- Static logit model attributes variance in market shares only to differences in product fixed effects, ignoring any dynamics.

U.S. Printer Market

- In 1998, printer sales accounted for 21% of total retail sales of computers and accessories, totaling $4.5 billion.

- In the five years after 1998, unit sales growth was 12%.

- Product differentiation in many dimensions such as:
  - Technology: inkjet vs. laser, color vs. black and white.
  - Functionality: printers vs. all-in-one devices.
  - Target sector: personal vs. workgroup.

- Highly concentrated industry with four-firm concentration ratio of 91% (HP, Epson, Canon, Lexmark).

Data Sources

- Primary source: *Hardware Monthly Report* by PC Data, Inc.
  - Consists of point-of-sale scaner level data on computer products sold by major retailers, mail order firms, and web sites.
  - This subset contains monthly sales figures and average prices of printers and multifunction devices.
  - Dates included: 14 months from January 1998 to February 1999.
  - Estimated market coverage: 55%.

- Secondary source: static printer characteristics from various sources such as CNET, CompUSA, and online documentation.

- Resulting sample: 462 models from 27 manufacturers.

- Descriptive statistics: Table 5.3, page 26.

Descriptive Statistics

- Between January 1998 to January 1999, average sales-weighted price of a printer fell by almost 16%. Lasers more expensive by 5% and inkjets less expensive by 20%.

- All technological factors such as speed, number of colors, and resolution have clear upward trends, especially installed RAM.

- Market share and revenue by speed and resolution (Figure 5.1, page 28).

- After controlling for quality, prices fell by 30%.

- Overall, market dynamics seem to be very important.

Estimation

- Parametric specification:

$$u_{jt} = \xi_j + x_j\beta - \alpha p_{jt} + \varepsilon_{jt}$$

$$r_{t+1} = r_t + \gamma + \sigma \nu_{t+1}$$

- M_t is taken to be the number of households with at least one computer.

- No instruments available for price, use fixed effects for price groups.

Empirical Results

- Static structural parameters: Table 5.6.
  - Most parameters behave as expected except RAM.
  - Strong negative effect for refurbished hardware.
  - Brand names appear to be very important.
  - Demand is most elastic for very low and very high price printers.

- Dynamic structural parameters: Table 5.7.
  - Strong support for forward-looking consumer behavior.
  - Replacement probability (0.02) implies printer lifetime of 4 years.
  - Two constrained models: myopic ($\beta = 0$) and patient ($\beta = 0.99$).

Conclusions

- Benefits of the model:
  - A first step towards endogenous reentry of consumers.
  - Quality and choice set may evolve over time.
  - Allows for intertemporal demand substitution.

- What's lacking:
  - Supply side: product introduction, entry, and exit.
  - Data: only retail sales, no institutional purchases.
  - Market quality continues to grow relative to outside good.
  - No secondary market.