Econ 301 **Problem Session 1**
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Problem 1. *Monotonicity of preference relations*

Show that

a. If $\succsim$ is strongly monotone, then it is monotone.

b. If $\succsim$ is monotone, then it is locally non-satiated.

Problem 2. *The strict dominance preference relation*

Let $X = \mathbb{R}^3_+$ be a choice set and let $x, y \in X$, where $x = (x_1, x_2, x_3)$ and $y = (y_1, y_2, y_3)$. Consider the *strict dominance* preference relation $\succsim$ on X, where

$$x \succsim y \quad \text{iff} \quad x_1 \geq y_1,\, x_2 \geq y_2, \quad \text{and} \quad x_3 \geq y_3.$$

Determine whether $\succsim$ is (i) complete, (ii) transitive, (iii) monotone, or (iv) convex.

Problem 3. *True or False?*

Determine whether the following statements are true or false and support your answer with a proof, counter-example, or rigorous reasoning.

a. Every rational preference relation on $X \subseteq \mathbb{R}$ has a utility function representation.

b. Every utility function induces a rational preference relation which represents the same preferences.

c. All utility functions representing convex preferences must be quasi-concave.

d. Rationality of preferences is sufficient to keep indifference curves from crossing.

Problem 4. *Lexicographic Preferences*

The Lexicographic preference ordering is defined for $x, y \in X = \mathbb{R}^2_+$ as follows: $x \succsim y$ iff $x_1 \geq y_1$ or ($x_1 = y_1$ and $x_2 \geq y_2$).

a. For some point $x \in X$, draw the upper and lower contour sets and the indifference curve at x.

b. Prove that there is no utility function representation for these preferences.

Problem 5. *Cobb-Douglas and Leontif Utility*

A consumer has a utility function of the form $u(x, y) = x^{\frac{1}{3}} y^{\frac{2}{3}}$. She has wealth w and faces prices p_x and p_y respectively for goods x and y.

a. Find her Marshallian (uncompensated) demand and her indirect utility function.

b. Now suppose the she has the utility function $u(x, y) = \ln x + 2 \ln y$. What are her Marshallian demand functions for both goods?

c. What about $u(x, y) = min\{x, 3y\}$?