

Econ 301 / Fall 2005
Problem Session 7

Problem 1. *MWG 5.C.1*

Prove that, in general, if the production set Y exhibits nondecreasing returns to scale, then either $\pi(p) \leq 0$ or $\pi(p) = \infty$.

Problem 2. *MWG 5.C.6*

Suppose $f(z)$ is a concave production function with $L-1$ inputs (z_1, \dots, z_{L-1}) . Suppose also that $\frac{\partial f(z)}{\partial z_l} \geq 0$ for all l and $z \geq 0$ and that the matrix $D^2 f(z)$ is negative definite at all z . Use the firm's first-order conditions and the implicit function theorem to prove the following statements:

- a. An increase in the output price always increases the profit-maximizing level of output.
- b. An increase in output price increases the demand for *some* input.
- c. An increase in the price of an input leads to a reduction in the demand for the input.

Problem 3. *MWG 5.D.5*

Suppose there are two goods: an input z and an output q . The production function is $q = f(z)$. We assume that f exhibits increasing returns to scale.

- a. Assume that f is differentiable. Do the increasing returns of f imply that the average product is necessarily nondecreasing in input? What about the marginal product?
- b. Suppose there is a representative consumer with the utility function $u(q) - z$ (the negative sign indicates that the input is taken away from the consumer). Suppose that $\bar{q} = f(\bar{z})$ is a production plan that maximizes the representative consumer utility. Argue, either mathematically or economically (disregard boundary solutions), that the equality of marginal utility and marginal cost is a necessary condition for this maximization problem.
- c. Assume the existence of a representative consumer as in b. "The equality of marginal cost and marginal utility is a sufficient condition for the optimality of a production plan." Right or wrong? Discuss.

Problem 4. *Duality in Production II*

Consider a firm that produces output (q) using capital (k) and labor (l). The firm's cost function is $c(r, w, q) = q^{3/2}(rw)^{1/2}$, where r is the price of capital and w is the wage rate.

- a. Derive the firm's production function from this cost function.
- b. Derive the supply curve for this firm for $r = w = 1$.