Econ 301 / Fall 2005
Problem Session 9

Problem 1. *Insurance*

Consider a strictly risk-averse individual with initial wealth w who may incurr a loss of D with probability π. The individual may purchase insurance at a price q that pays 1 in the event of a loss and nothing otherwise. Assume the agent's utility function over wealth is $u(w) = w$.

a. Suppose insurance is actuarially fair. Set up the individual's utility maximization problem and show that he fully insures.

b. Show that if the price of insurance is higher than the actuarially fair price, the individual will not insure completely.

Problem 2. *True or False*

a. If the distribution F first-order stochastically dominates G, then no rational individual with an increasing Bernoulli utility function will ever choose the lottery represented by G.

b. If the distribution F second-order stochastically dominates G, then no rational individual with an increasing Bernoulli utility function will ever choose the lottery represented by G.

c. Suppose that two agents have identical risk-averse preferences and identical wealth. They hold two assets with identical return distribution but independent realizations. Allowing for asset trade will not result in Pareto improvement.

d. If the preference relation $\succsim$ on L satisfies the independence axiom, then $L \sim \alpha L' + (1-\alpha)L''$ for every $\alpha \in [0,1]$ implies that $L = \alpha L' + (1-\alpha)L''$ for some α.

e. Suppose that the utility function of each outcome $(u_1, \ldots, u_n)$ on a set C is equal to $\bar{u}$. The individual will be indifferent between any lotteries defined over this set of outcomes.

Problem 3. *CARA*

Suppose Leo is a risk-averse, expected utility maximizer with monotone increasing utility over money. His initial wealth is w. Leo can choose between the following lotteries:

- $L_1 = \left(\frac{1}{2}, \frac{1}{2}\right)$ over outcomes (win \$200, lose \$100),
- L_2: win nothing for sure, lose nothing for sure,
- L_3: win \$100 for sure,

- $L_4 = \left(\frac{1}{2}, \frac{1}{2}\right)$ over outcomes (win \$300, lose nothing),
- $L_5 = (p, 1-p)$ over outcomes (win \$300, lose \$100).

Leo says he is indifferent between L_1 and L_2 as well as L_3 and L_4.

a. What is Leo's certainty equivalent for L_1? For L_4?

b. For which values of p can you be sure Leo will prefer L_5 to L_2?

c. True or False: Any rational agent with CARA preferences who is indifferent between L_1 and L_2 is also indifferent between L_3 and L_4.

d. Suppose Leo has CARA utility with a coefficient of absolute risk-aversion of λ. What is his probability premium? Does it depend on w? Explain why or why not.

Problem 4. *CRRA*

Consider the utility function

$$u(x) = \frac{x^{1-\sigma}}{1-\sigma}.$$

a. Show that this function exhibits risk aversion and implies constant relative risk aversion.

b. Show that $u(x) = \ln(x)$ is a special case of the above utility function with $\sigma = 1$.

c. Assume that individuals have initial wealth w and they must invest a proportion of their wealth α in a risky asset A with returns distributed as $U(0, 4)$. Show that α does not depend on w and decreases with σ.

d. Now, consider another risky asset B whose returns have a triangle distribution on $[0, 4]$, with density function

$$f(x) = \begin{cases} \frac{1}{4}x & \text{if } 0 \leq x \leq 2 \\ 1 - \frac{1}{4}x & \text{if } 2 < x \leq 4 \\ 0 & \text{otherwise} \end{cases}.$$

Show that B second order stochastically dominates A.

e. Show that for any σ the agent will invest a higher proportion of his wealth in B than in A.