Econ 302 **2006.02.24**
Problem Session 6 **Jason Blevins**

Problem 1. *Simple Bargaining Model*

A seller is selling an object which he values at $v_S \sim F_S(v) = v^2$ on $[0, 1]$. F_S is common knowledge, but the realization of v_S is known only to the seller. A buyer, who values the object at $v_B = 1$, is given the chance to pay c in order to make a take-it-or-leave-it offer or quit without making an offer and end the game. After the buyer makes an offer, the seller can either accept or reject the offer. Find the pure strategy Nash equilibrium.

Problem 2. *Question 2, Midterm 2005*

Consider a discrete-time war of attrition between two risk-neutral players. Each player has fighting cost $c > 0$ and values the prize at $v \in (c, 2c)$. At the beginning of each period the players simultaneously decide whether to *fight* or *quit*. The game ends the first time one or both players choose to quit. The prize is awarded to a player when his rival quits and he does not. The winner must pay his fighting cost in the last period (i.e., in the period when his rival quits). There is no discounting (i.e., $\delta = 1$). Index time by $t = 1, 2, \dots$. Hence, if player i quits at date τ, then his payoff is $-(\tau - 1)c$. If player i quits at date τ and player j does not, then j's payoff is $v - \tau c$. Here is the tricky part. Player 1 has limited resources and can fight for at most $T > 1$ periods while player 2 has no such constraint.

a. Find the non-degenerate mixed-strategy probability that each player quits at date T, $m_1(T)$ and $m_2(T)$.

b. Which player quits with higher probability at date T? Prove your answer and provide brief intuition.

c. Find the non-degenerate equilibrium probabilities of quitting at date $T-1$, $m_1(T-1)$ and $m_2(T-1)$.

d. Specify the SPNE strategies for each player corresponding to a non-degenrate mixed-strategy equilibrium. Which player is more likely to win the war in this SPNE?

e. Is the SPNE you found in part d an MPE? Explain.

Problem 3. *Question 4, Midterm 2003*

Consider the following principal-agent problem. The agent chooses between two levels of effort, $a_L = 0$ and $a_H = 1$. The cumulative distribution functions of profits are:

$$F(\pi \mid a_L) = \begin{cases} 0, & \text{if } \pi < 0 \\ 1/4 + 3\sqrt{\pi}/8, & \text{if } \pi \in [0, 4) \\ 1, & \text{if } \pi \geq 4 \end{cases}$$

and

$$F(\pi \mid a_H) = \begin{cases} 0, & \text{if } \pi < 0 \\ \pi^2/16, & \text{if } \pi \in [0,4) \\ 1, & \text{if } \pi \geq 4. \end{cases}$$

The principal is risk-neutral. The agent's reservation utility $\bar{u} = -1$ and his preferences over wages $w \in \mathbb{R}$ and effor $a \in \{a_L, a_H\}$ are represented by

$$U(w,a) = -\exp\{-(w-a)\}.$$

a. Draw $F(\pi \mid a_L)$ and $F(\pi \mid a_H)$. Are the distributions rankable in the sense of first-order stochastic dominance? Does the monotone likelihood ratio property (MLRP) hold?

b. Derive the full-information first-best contract, $(a^*, w^*(\cdot))$. How much expected *net* profit, $\mathrm{E}[\pi - w^*(\pi) \mid a^*]$, does the principal make under the first-best contract?

c. Now suppose effort is not verifiable. Derive the optimal (second-best) compensation schedule, $w^{**}(\pi)$, in this case. Denote by a^{**} the effort it induces.

d. How much expected *net* profit, $\mathrm{E}[\pi - w^{**}(\pi) \mid a^{**}]$, does the principal make under the second-best contract? Compute the agency costs.