Problem 1. EV/CV

Jim gets utility only from lemonade made according to his family recipe. He gets 3 units of utility from 1 glass of lemonade. Each glass of lemonade requires 4 lemons and 1 cup of sugar. Jim has $6. Lemons cost $0.10 each and the price of sugar just fell from $0.80 to $0.60 per cup.

a. Find an expression for Jim’s utility.

b. Calculate the EV and CV.

Problem 2. Fall 2003, Part 1, Question 1

Suppose that during the hot summer, Joe consumes only cola ($x$) and watermelon ($y$). His preferences are represented by the following utility function

$$u(x, y) = \ln x + \ln y.$$ 

Joe has wealth $w > 0$ and the prices of cola and watermelon are $p_x > 0$ and $p_y > 0$ respectively.

a. Solve Joe’s utility maximization problem and derive his Walrasian demand for cola and watermelon.

b. Solve Joe’s expenditure minimization problem and derive his Hicksian demand for cola and watermelon.

c. Show that his Walrasian demand curve for cola is steeper (in $p_x$) than the Hicksian one for $p_x = p_y = 1$ and $\bar{u} = v(1, 1, w)$. Why is it so?

Now assume that Joe actually does not know the price of watermelon when he has to decide how much cola he wants to buy. In particular he knows only that the price of watermelon is either $p_1$ or $p_2$ with equal probabilities. [Hint: Assume that he has to spend all of his remaining money on watermelon.]

d. Derive his demand for cola.

e. Evaluate the following statement: “Assume a situation when $\frac{1}{2}p_1 + \frac{1}{2}p_2 = p_y$. Then the uncertainty Joe faces is a mean preserving spread around $p_y$. Therefore, since Joe is risk averse, he will buy more cola in the risky situation described above compared to the case when he faces a certain $p_y$ price of watermelon.”
Problem 3. Fall 2002, Part 1, Section 2, Problem 2
Suppose you are at the mall and need to go up a floor. You have in front of you two options: stairs and an escalator (or “moving stairs”). The escalator travels at a speed $e$ (in units of distance per unit of time), but there is a line $q$ minutes long to use it. There is no line to use the stairs. The distance to the next floor is $h$ for both the escalator and stairs. Your time is worth $c$ per unit of time. On either the escalator or the stairs, you may walk at the speed $w \in [0, \infty)$ of your choosing. (Thus on the escalator your total speed would be $e + w$.) For each unit of time spent walking, the cost is $w^2$. Be sure to explain your answers for each of the following.

a. If you take the stairs, at what speed $w_s$ do you walk and how long does it take? How does the length of the line affect your speed and total time?

b. If you take the elevator, at what speed $w_e$ do you walk? How does the speed of the elevator affect your walking speed? How does the length of the line affect your speed and total time?

c. Do you walk faster on the escalator or stairs? On which is your total speed greater?

d. If everyone has the same value of time and cost of walking, what should the length of the line be in terms of $w_e$ and the parameters of the model? If the distance $h$ were greater, what happens to the length of the line?

e. It is observed that sometimes people stand still on the escalator. What is a simple modification of this model that would give rise to such a result? (You do not need to formally demonstrate this.)

Problem 4. Pizza and beer
Suppose both you and I have identical preferences and incomes and both of us consume only beer and pizza. In fact, the only difference between us is that I own a coupon that allows me to buy pizza at one-fourth of the regular price. You come to me to see if we can agree on a price at which I would sell the pizza coupon to you.

a. Is there a price that would be mutually acceptable to both of us? What does your answer depend on? [Use the concepts of equivalent and compensating variation to answer this problem.]

b. Suppose $x$ stands for pizza, $y$ stands for beer, and $w$ is income. Our (Walrasian) demand function for pizza is $x(p_x, p_y, w) = (w + 3p_y)/2p_x$ and our expenditure function is $e(p_x, p_y, \bar{u}) = 2\bar{u}\sqrt{p_x p_y} - 3p_y$. When you are trying to make a deal with me, the price of pizza is four, the price of beer is one and our income is 37 each. How much would you be willing to pay me for the coupon? What is the least I am willing to accept? Given your answer to part a, is pizza an inferior good for us?